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Black Hole Atom

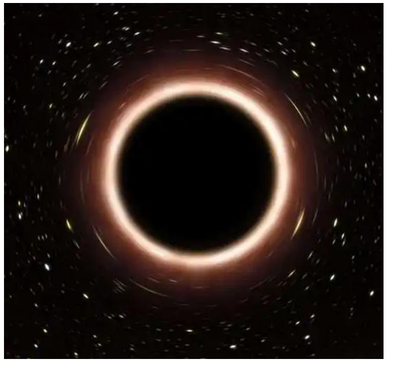
张宏升(Hongsheng Zhang)
Co-Author：黄洋(Yang Huang)
济南大学(UJN)

Principle refs：2006.01388，2012.12778，2109.05876，2206.05645

Outline

- Atom VS black hole
- Dilatonic(GM) black hole
- Bound state
- Scattering state
- Of the horizon singularity
- Summary

Atom VS black hole: No-hair



particle: mass, spin, charges etc

black hole: mass, spin, charge (electric)

Particle VS black hole: Entropy

- Elementary particle: Zero (von Neumann)
- Black hole: Large (Boltzmann);

  Arbitrary? determined by initial states(von Neumann)

Particle VS black hole: Temperture

- Elementary particle: meaningless
- Black hole: finite (implies internal freedoms)

Particle VS black hole: Scattering

- Elementary particle: elastic scattering state
- Black hole: no elastic scattering state

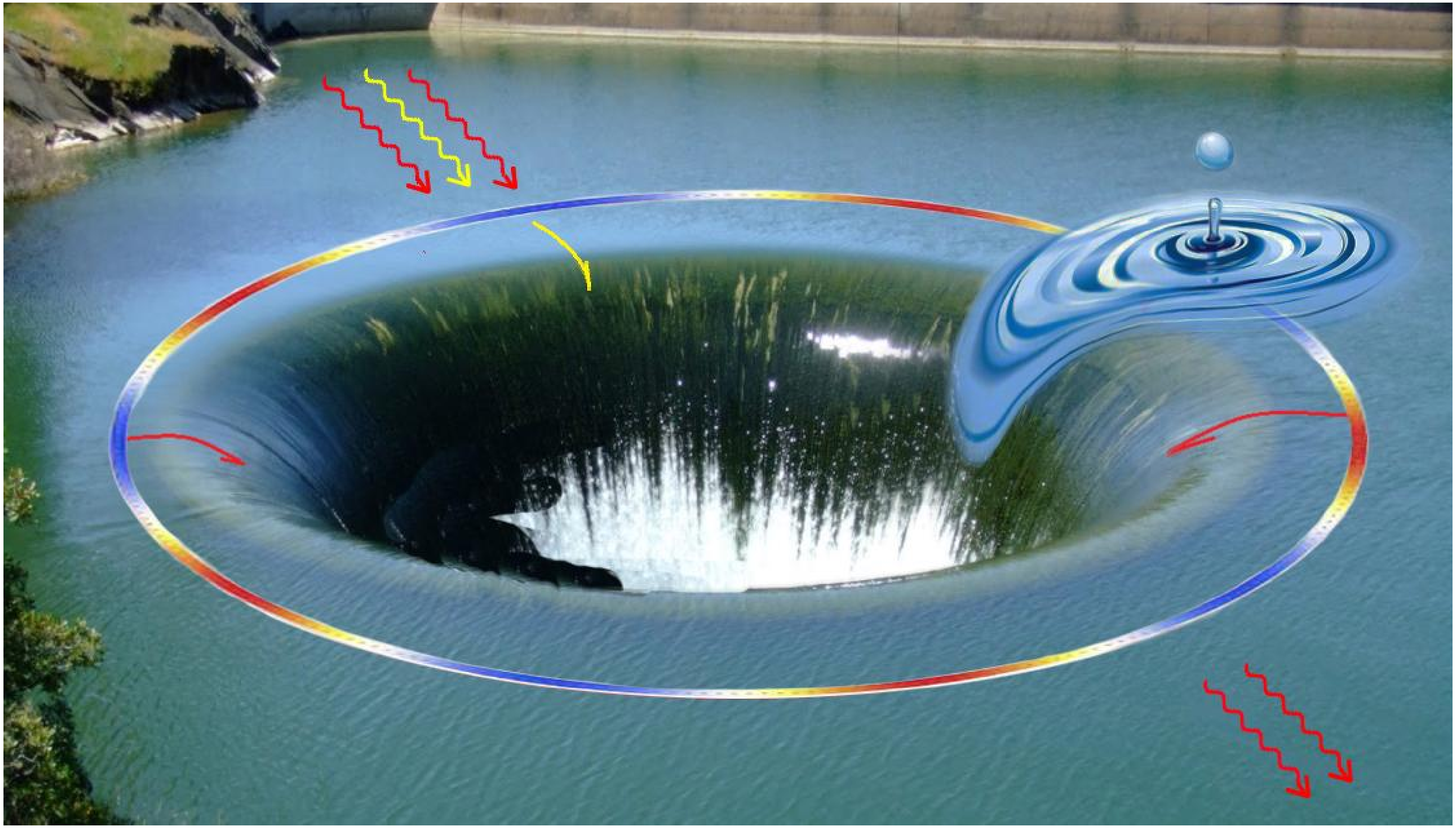
Particle VS black hole: Bound state

- Elementary particle: stable
- Black hole: unstable because of leakage into the interior of the hole (implies internal freedoms)

Particle VS black hole: QNM

- Elementary particle: No
- Black hole: Important

Three states for BHs



感谢闫慧同学帮助制图

Extreme holes: RN

- T=0 (can still has internal freedoms)
- $S=\pi M^2$ (large internal freedoms)
- Bound state: leakage
- Scattering state: Inelastic
- QNM: exist

- HOW About Dilatonic Black Holes?

Dilatonic BH (GMGHS)

$$S = \int d^4x\sqrt{-g}[R - 2(\nabla\phi)^2 - e^{-2a\phi}F_{\mu\nu}F^{\mu\nu}]$$

$$ds^2 = -Fdt^2 + F^{-1}dr^2 + r^2G(d\vartheta^2 + \sin^2\vartheta d\varphi^2)$$

$$F(r) = 1 - \frac{2M}{r}, \quad \text{and} \quad G(r) = 1 - \frac{Q^2}{Mr}$$

$$F_M = Q\sin\vartheta d\vartheta \wedge d\varphi$$

$$e^{-2\phi} = e^{-2\phi_0}\left(1 - \frac{Q^2}{Mr}\right)$$

Status: 1. old, Dirac large number hypothesis, KK theory. 2. the minimal modified gravity (only one extra freedom). 3. can be reduced from several other theories.

G.W. Gibbons and K.-i. Maeda, Nucl. Phys. B298, 741(1988)；D. Garfinkle, G. T. Horowitz, and A. Strominger, Phys. Rev. D 43, 3140 (1991); 45, 3888(E) (1992).

Thermodynamics of Dilatonic BH

$$S = \pi r_+^2 \left(1 - \frac{r_-}{r_+}\right)$$

$$M = \frac{r_+}{2}$$

$$Q^2 = r_+ r_- / 2.$$

Extreme dilatonic BH

- Residue supersymmetry->freedom count in string theory
- S=0, which implies
- no internal freedom, such that it may become an elementary particle (conjectured in C Holzhey and F Wilczek,1992; J Preskill, P Schwarz, A Shapere, S Trivedi, F Wilczek, 1991).
- We propose,
- elastic scattering
- stable bound sate
- Nomal (Without quasi-) modes
- etc.

Dilatonic hole (a=1): the enigma

for $a < 1$, and elementary point objects for $a > 1$. Unfortunately the case suggested by superstring theory, $a = 1$, is enigmatic. However, we can hardly refrain from

by C Holzhey and F Wilczek,1992

Bound state: boundary conditions

- At horizon: $\psi_{\omega l} \sim e^{-i\omega x} \sim (r - r_+)^{-2iM\omega}$.
- x: Toi Cor
- At spatial infinity: $\psi_{\omega l} \sim r^{\chi} e^{\rho r}$
- $$\chi = \frac{M(\mu^2 - 2\omega^2)}{\rho}, \quad \text{with} \quad \rho = \pm\sqrt{\mu^2 - \omega^2}.$$

Near horizon Aproximation

$$z \equiv \frac{r - r_+}{r_+}; \qquad \tau \equiv \frac{r_+ - r_-}{r_+} = 1 - q^2.$$

$$z(z+\tau)\frac{d^2R}{dz^2} + (2z+\tau)\frac{dR}{dz} + VR = 0,$$

$$V = \frac{4\tau\epsilon^2}{z} - l(l+1) + 4(1+2\tau)\epsilon^2 - 4\tau\mu_s^2$$
$$+ 4[\epsilon^2 + (1+\tau)(\epsilon^2 - \mu_s^2)]z + 4(\epsilon^2 - \mu_s^2)z^2.$$

$\epsilon = \omega M$ and $\mu_s = \mu M$

Near horizon Aproximation

$$z(z+\tau)\frac{d^2R}{dz^2}+(2z+\tau)\frac{dR}{dz}+\left(\frac{4\tau\epsilon^2}{z}+\frac{1}{4}-\beta^2\right)R=0,$$

$$\beta^2=\left(l+\frac{1}{2}\right)^2+4\tau\mu_s^2-4(1+2\tau)\epsilon^2$$

$$R(z)\sim\frac{\Gamma(1-4i\epsilon)\Gamma(2\beta)}{\Gamma(\frac{1}{2}+\beta-2i\epsilon)^2}\left(\frac{z}{\tau}\right)^{-\frac{1}{2}+\beta}+(\beta\to-\beta)$$

Far field approximation

$$z^2 \frac{d^2R}{dz^2} + 2z\frac{dR}{dz} + \left[\frac{1}{4} - \beta^2 + 2\kappa k z - k^2 z^2\right] R = 0 \qquad \kappa = \frac{4\epsilon^2 - (1+\tau)k^2}{2k}$$

$$k = 2\sqrt{\mu_s^2 - \epsilon^2}$$

$$R(z) = C_1 e^{-kz}(2k)^{\frac{1}{2}+\beta} z^{-\frac{1}{2}+\beta} M\left(\frac{1}{2} + \beta - \kappa, 1 + 2\beta, 2kz\right) + C_2 \times (\beta \to -\beta), \qquad (\text{:}$$

Matching condition

$$\frac{\Gamma(\frac{1}{2}-\beta-\kappa)}{\Gamma(\frac{1}{2}+\beta-\kappa)} = \frac{\Gamma(\frac{1}{2}+\beta-2i\epsilon)^2\Gamma(-2\beta)^2}{\Gamma(\frac{1}{2}-\beta-2i\epsilon)^2\Gamma(+2\beta)^2}(2k\tau)^{2\beta}.$$

$$\frac{1}{2}+\beta-\kappa=-n \qquad\qquad \epsilon=\mu_s\left[1+\sum_{i=1}^{\infty}C_i\mu_s^{2i}\right]$$

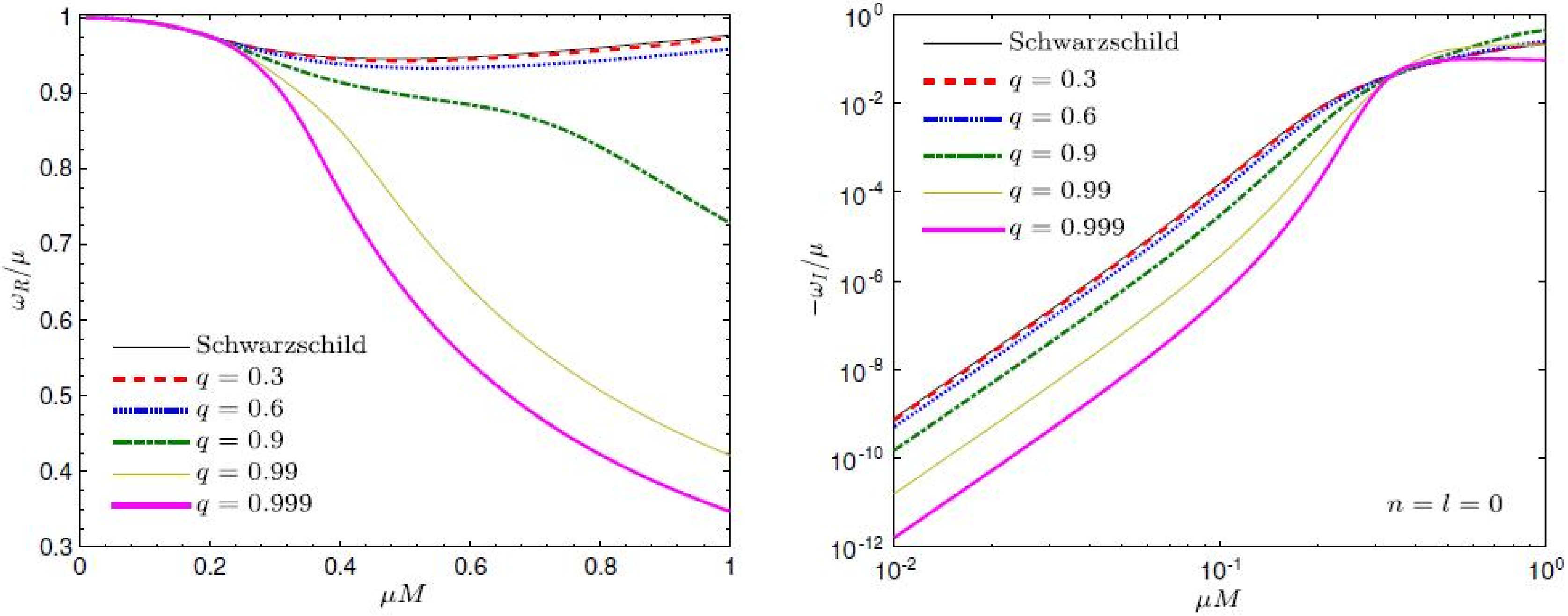
$$C_1=-\frac{1}{2\tilde{n}^2},$$

$$C_2=-\frac{2(1+\tau)}{\tilde{n}^3L}+\frac{\tau+15/8}{\tilde{n}^4},$$

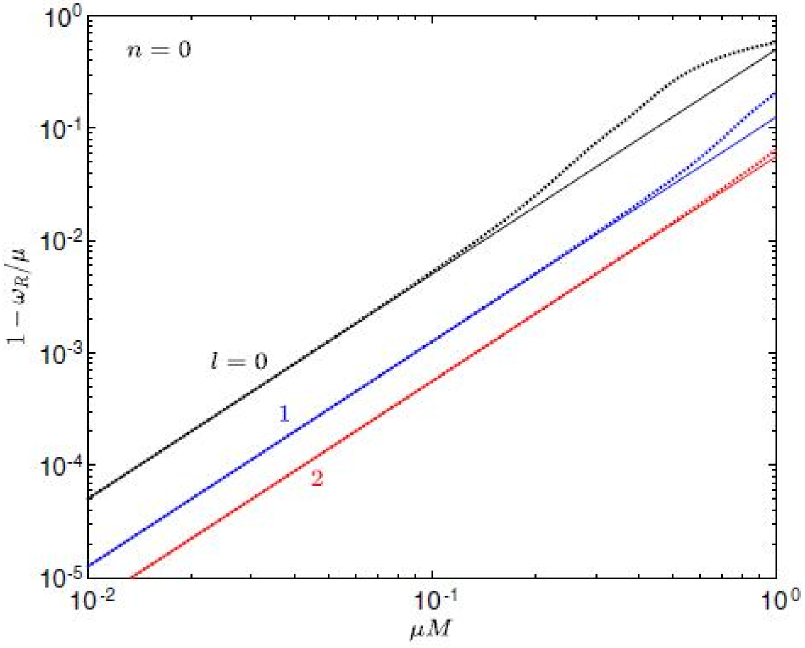
$$C_3=-\frac{2(1+\tau)^2}{\tilde{n}^3L^3}+\frac{6(\tau+1)^2}{\tilde{n}^4L^2}$$

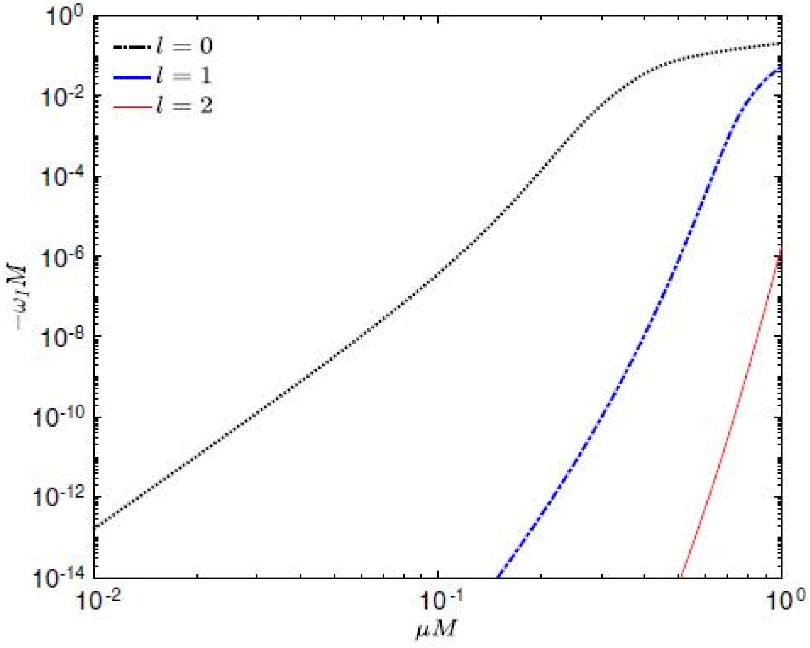
$$+\frac{8\tau^2+27\tau+17}{\tilde{n}^5L}-\frac{40\tau^2+152\tau+145}{16\tilde{n}^6}. \qquad \tilde{n}=n+l+1 \ \text{ and } \ L=l+1/2$$

Numerical results



Analytical VS numerical results





Exact bound solution for extreme holes

$$\frac{d^2\psi}{dz^2}+\frac{1}{z(z+1)}\frac{d\psi}{dz}-U\psi=0$$

$$z=\frac{r-r_+}{r_+};\quad \epsilon=\omega M;\quad \alpha=\mu M.$$

$$U(z)=\frac{4(1+z)}{z}\left[\alpha^2-\left(1+\frac{1}{z}\right)\epsilon^2\right]+\frac{l(l+1)}{z^2}+\frac{1+4z}{4z^2(1+z)^2}.$$

$$\psi(z)=z^\beta(1+z)^{-1/2}e^{-kz}L^{2\beta}_{-1/2-\beta+\kappa}(2kz),$$

$$k=2\sqrt{\alpha^2-\epsilon^2},\ \kappa=(4\epsilon^2-k^2)/(2k)\qquad \beta=\sqrt{L^2-4\epsilon^2},\quad \text{with}\quad L\equiv l+1/2.$$

Energy spectrum

$$-\frac{1}{2} - \beta + \kappa = n$$

$$\epsilon_n = \alpha\left[1 + \sum_{i=1}^{\infty} C_i \alpha^{2i}\right]$$

$$C_1 = -\frac{1}{2\tilde{n}^2},$$

$$C_2 = -\frac{2}{\tilde{n}^3 L} + \frac{15}{8\tilde{n}^4},$$

$$C_3 = -\frac{2}{\tilde{n}^3 L^3} - \frac{6}{\tilde{n}^4 L^2} + \frac{17}{\tilde{n}^5 L} - \frac{145}{16\tilde{n}^6}$$

$$\tilde{n} = n + l + 1$$

Bound state: Dirac particles

$$\left[\gamma^{\alpha}\left(\partial_{\alpha}-\Gamma_{\alpha}\right)-\mu\right]\mathbf{\Psi}=0 \qquad \Gamma_{\alpha}=-\frac{1}{4}g_{\mu\nu}e_{a}^{\mu}e_{b;\alpha}^{\nu}\hat{\gamma}^{a}\hat{\gamma}^{b} \qquad \gamma^{\mu}=e_{a}^{\mu}\hat{\gamma}^{a}$$

$$\Gamma_{t}=\frac{F'}{4}\hat{\gamma}^{0}\hat{\gamma}^{1}, \qquad \Gamma_{r}=0,$$

$$\Gamma_{\vartheta}=W\,\hat{\gamma}^{1}\hat{\gamma}^{2},$$

$$\Gamma_{\varphi}=W\sin\vartheta\,\hat{\gamma}^{1}\hat{\gamma}^{3}+\frac{\cos\vartheta}{2}\hat{\gamma}^{2}\hat{\gamma}^{3} \qquad W(r)=\frac{r}{4}\sqrt{\frac{F}{G}}\left(\frac{2}{r}+\frac{d}{dr}\right)G$$

$$ds^{2}=-F dt^{2}+F^{-1}dr^{2}+r^{2}G\left(d\vartheta^{2}+\sin^{2}\vartheta d\varphi^{2}\right)$$

Hamiltonian

$$\mathbf{\Phi} = rG^{1/2}F^{1/4}\left(\sin\vartheta\right)^{1/2}\mathbf{\Psi}$$

$$H = i\hat{\gamma}^0\hat{\gamma}^1 F\partial_r + \frac{1}{r}\sqrt{\frac{F}{G}}\hat{\gamma}^1 K - i\mu\sqrt{F}\hat{\gamma}^0,$$

$$K = i\hat{\gamma}^1\hat{\gamma}^0\hat{\gamma}^2\partial_\vartheta + i\hat{\gamma}^1\hat{\gamma}^0\hat{\gamma}^3\csc\vartheta\partial_\varphi.$$

$$[K, H] = 0$$

$$\Phi_\mu = R_\mu(r)\Theta_\mu(\vartheta,\varphi)e^{-i\omega t}$$

Four-spinor

$$R = \begin{pmatrix} R_+ \\ R_- \end{pmatrix}$$

$$\mathscr{D}_\pm R_\pm = \mathscr{C}_\mp R_\mp \qquad \mathscr{D}_\pm = F\partial_r \pm i\omega$$

$$\mathscr{C}_\pm = \frac{1}{r}\sqrt{\frac{F}{G}}\left(\lambda \pm i\mu r\sqrt{G}\right)$$

Boundary conditions

At horizon

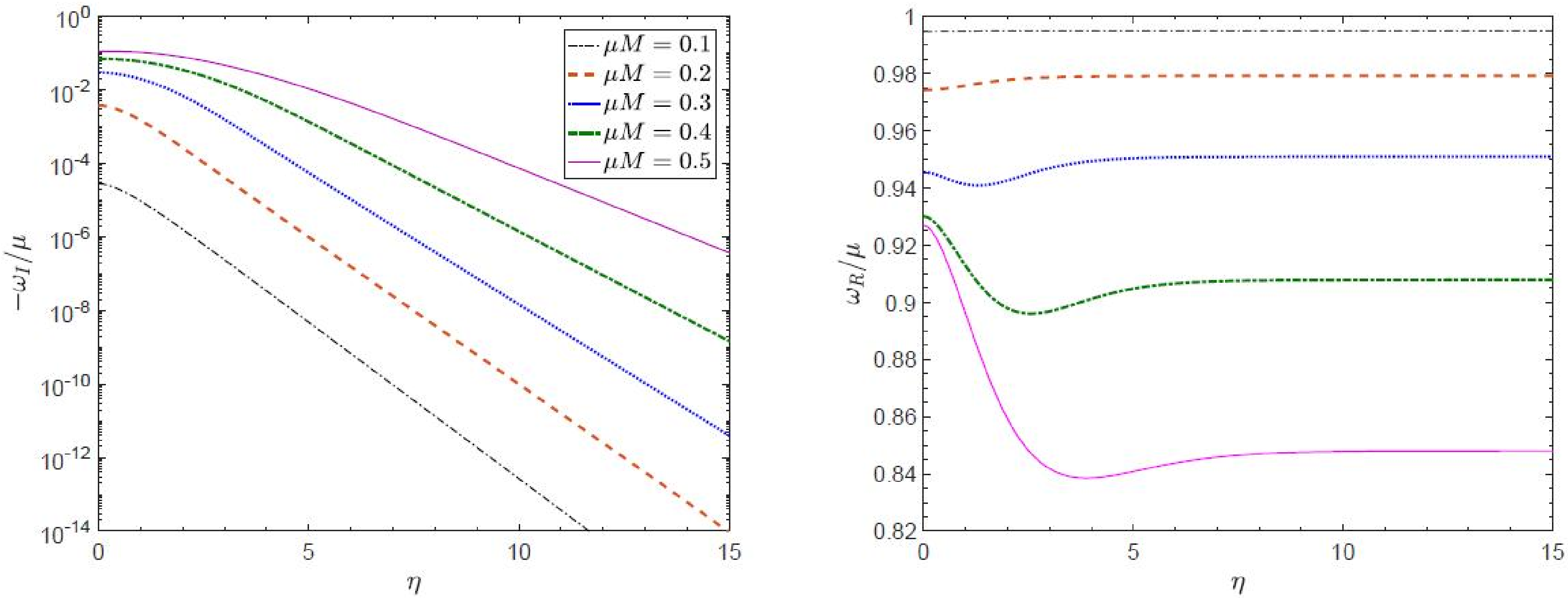
$$R_\pm \sim \mathcal{T}_\pm (r - r_+)^{\frac{1\mp 1}{4}} e^{-i\omega r_*}$$

At infinity

$$R_\pm \sim r^\chi e^{kr}$$

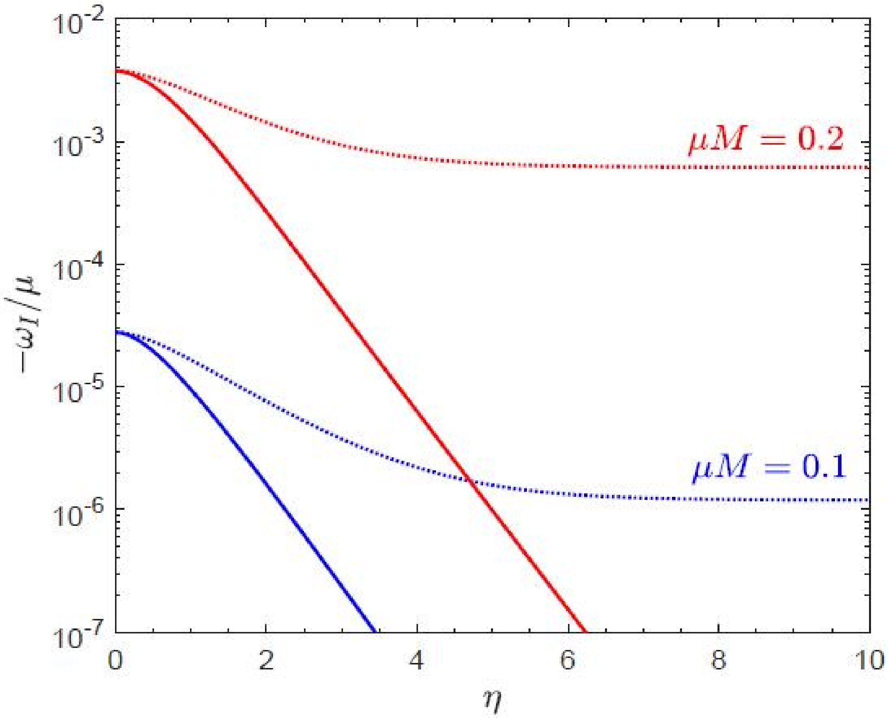
$$k = -\sqrt{\mu^2 - \omega^2}, \text{ and } \chi = M(\mu^2 - 2\omega^2)/k$$

Eigen frequencies

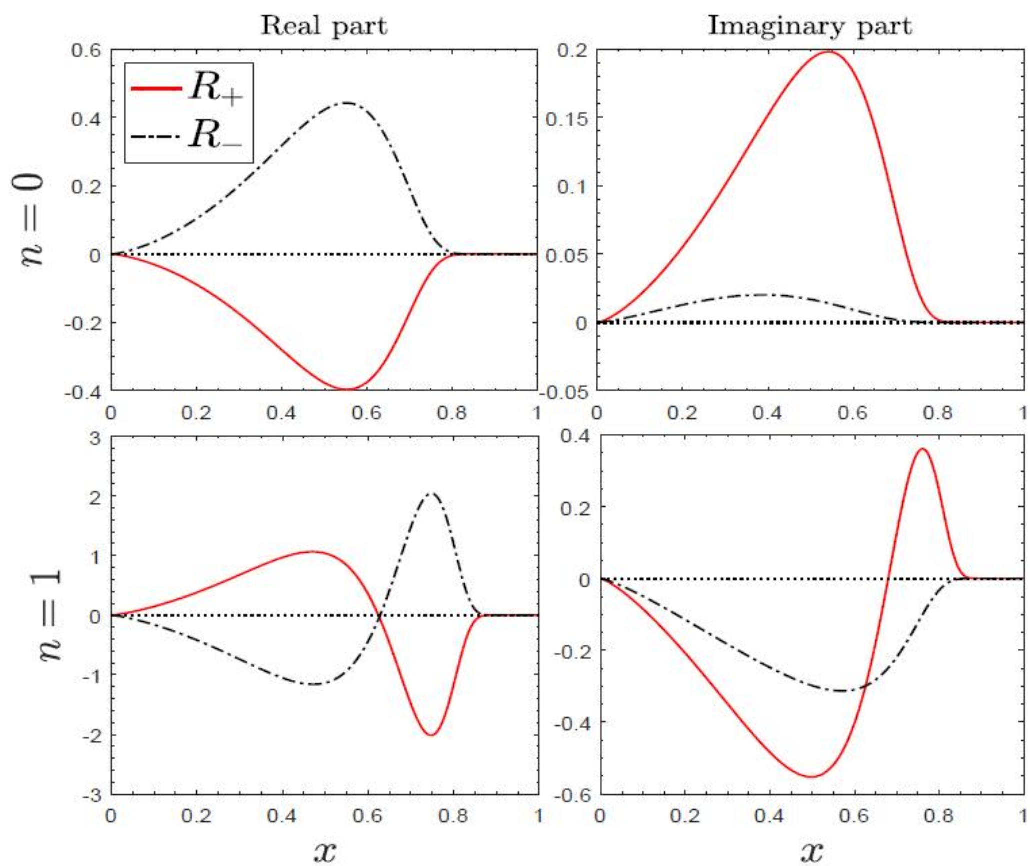


$$q = Q/Q_{\max} = 1 - e^{-\eta}$$

Compared to RN

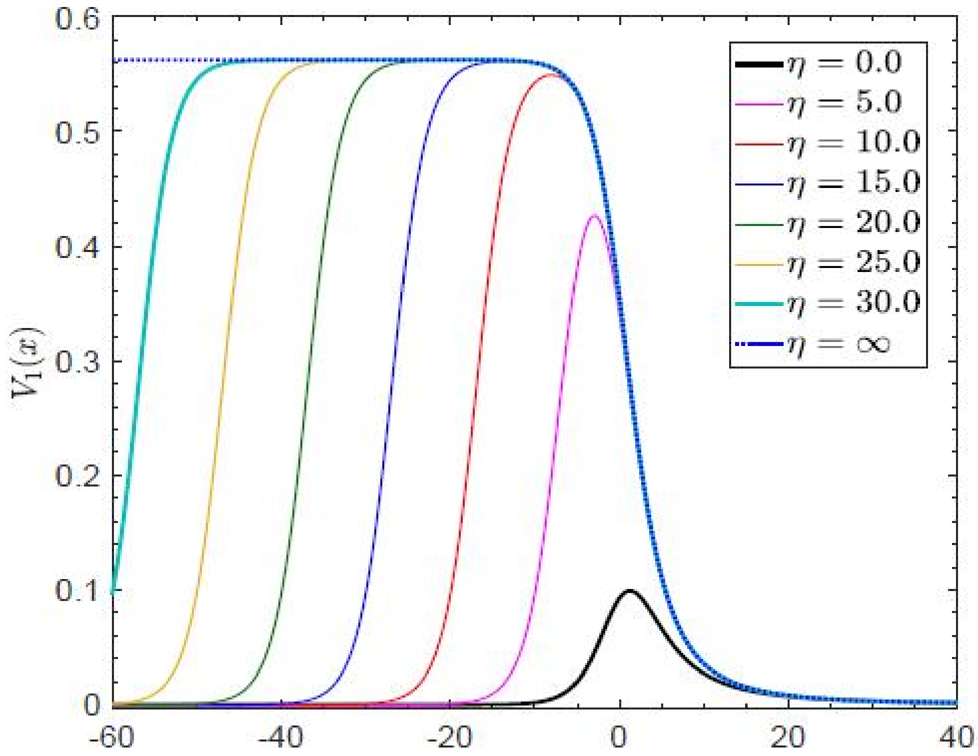


Wave functions



$$x \equiv \frac{\sqrt{(r-r_+)\mu}}{\sqrt{(r-r_+)\mu}+1} \in [0,1]$$

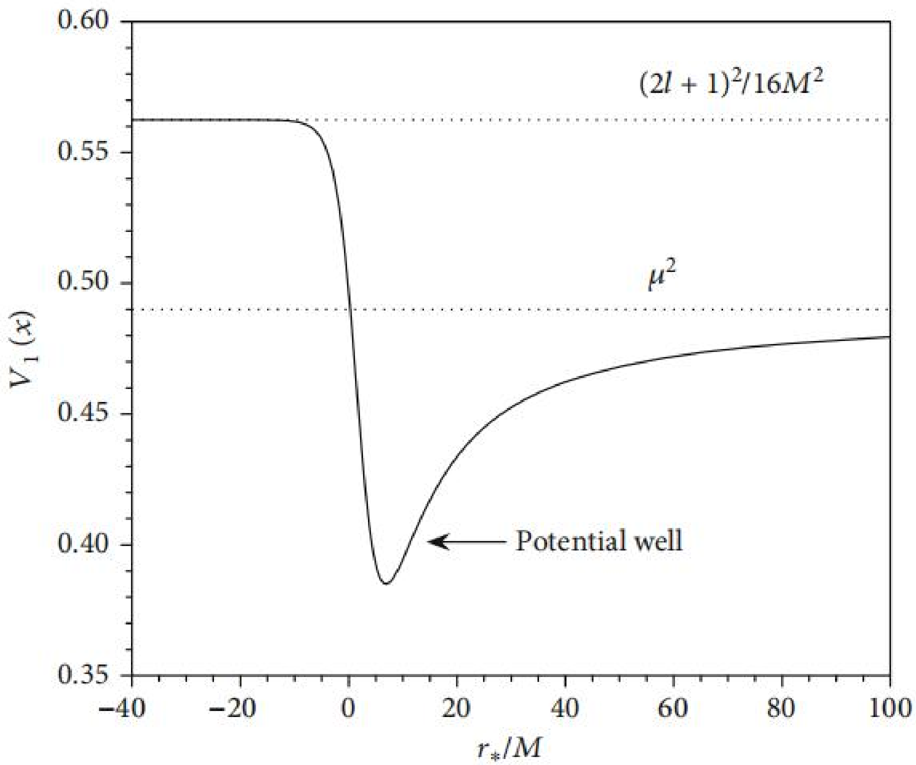
Physical nature of bound states: the effective potential



$l = 1$ and $\mu M = 0$

$$q = Q/Q_{\max} = 1 - e^{-\eta}$$

Potential well for massive particles



Wave scattering: fundamental equations

$$\nabla_\mu \nabla^\mu \Phi = 0.$$

$$\Delta \frac{d}{dr}\left(\Delta \frac{dR_{\omega l}}{dr}\right) + \left[G(r)^2\omega^2 r^4 - \Delta l(l+1)\right] R_{\omega l} = 0,$$

Δ $=(r-r_+)(r-r\_)$

$$\frac{d^2}{dr_*^2}\psi_{\omega l}(r) + \left[G(r)^2\omega^2 - V_l(r)\right]\psi_{\omega l}(r) = 0,$$

$$V_l(r) = f(r)\left[\frac{f'(r)}{r} + \frac{l(l+1)}{r^2}\right]$$

Wave scattering: boundary conditions

Near horizon: $\psi(r) \sim e^{-i\kappa\omega r_*}$

Spatial infinity: $\psi(r) \sim A_l^{(-)}(\omega)e^{-i\omega r_*} + A_l^{(+)}(\omega)e^{i\omega r_*}$.

Element of S-matrix: $S_l(\omega) = e^{i(l+1)\pi}\frac{A_l^{(+)}(\omega)}{A_l^{(-)}(\omega)}$.

Poles: (i) for $l \in \mathbb{N}$ (ii) Regge pole

poles of complex ω: QNM poles of complex l :Scattering

Wave scattering: Regge pole approach

$$\frac{d\sigma}{d\Omega} = |f(\omega, \theta)|^2.$$

$$f(\omega, \theta) = \frac{1}{2i\omega} \sum_{l=0}^{\infty} (2l+1) [S_l(\omega) - 1] P_l(\cos\theta)$$

partial wave

$$f_{\mathrm{P}}(\omega, \theta) = -\frac{i\pi}{\omega} \sum_{n=1}^{\infty} \frac{\lambda_n(\omega) r_n(\omega)}{\cos[\pi \lambda_n(\omega)]} P_{\lambda_n(\omega)-1/2}(-\cos\theta).$$

Regge pole

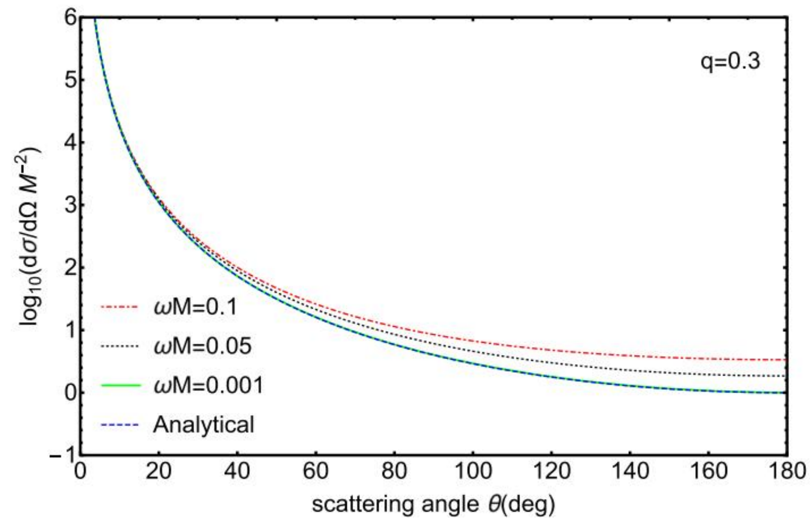
$$r_n(\omega) = e^{i\pi[\lambda_n(\omega)-1/2]} \left[\frac{A^{(+)}_{\lambda-1/2}(\omega)}{\frac{d}{d\lambda} A^{(-)}_{\lambda-1/2}(\omega)} \right]_{\lambda=\lambda_n(\omega)}$$

Residue of S

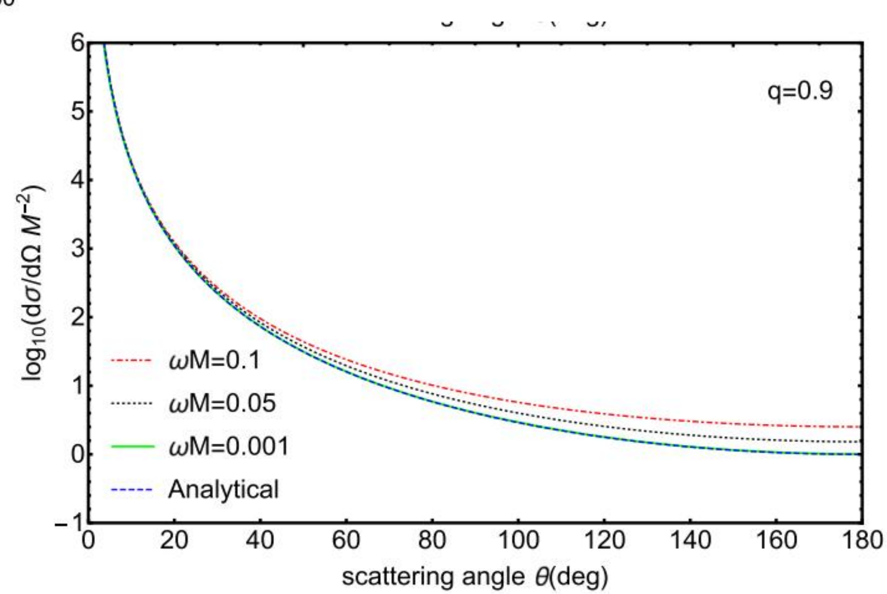
$$\frac{d}{d\lambda} A^{(-)}_{\lambda-1/2} = \frac{A^{(-)}_{\lambda_n+\delta-1/2} - A^{(-)}_{\lambda_n-1/2}}{\delta}$$

MST method, and continued fraction to find poles. delta=10^(-7)
MST, arXiv:gr-qc/9603020

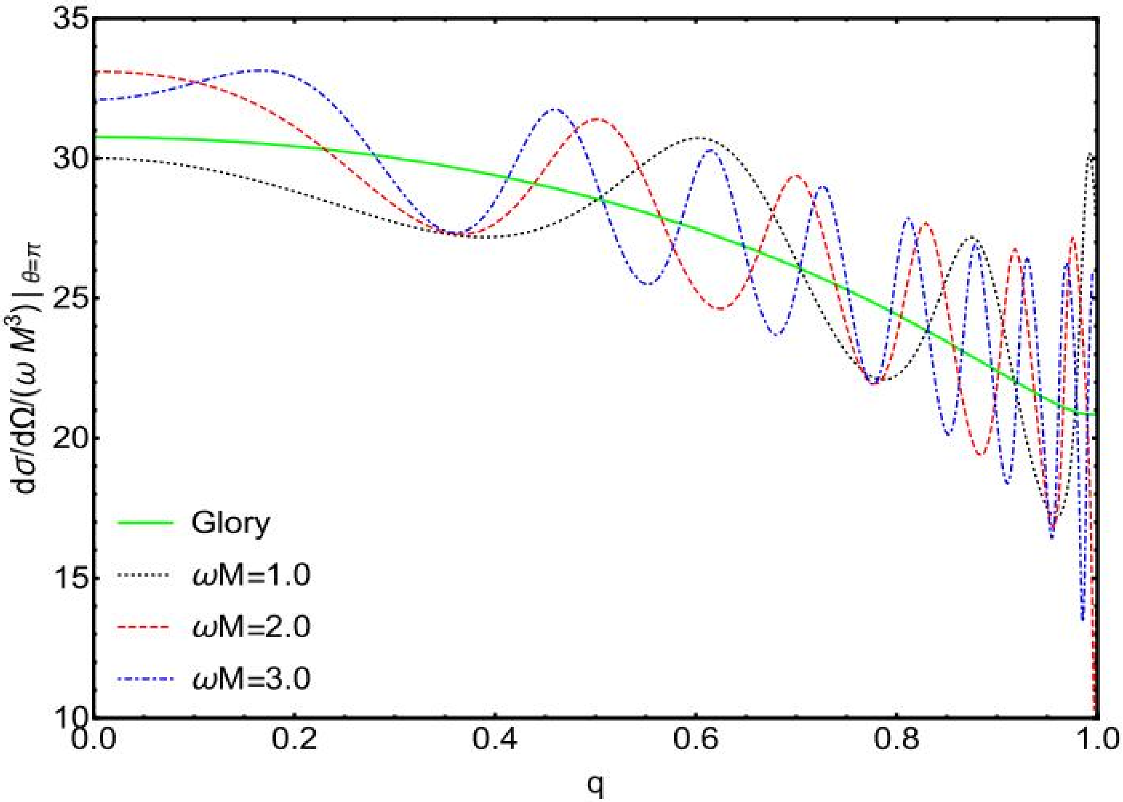
The low frequency behavior of the scattering cross section for q = 0.3 (top) and 0.9 (bottom).



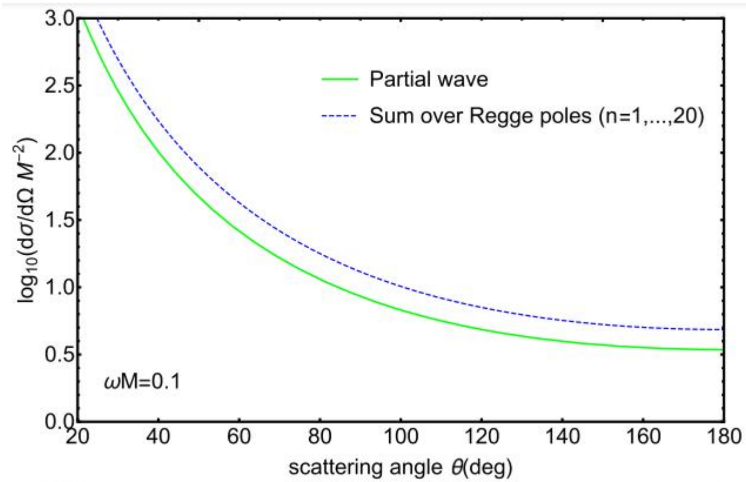
$$\lim_{\omega M \to 0} \left(\frac{1}{M^2} \frac{d\sigma}{d\Omega} \right) = \frac{1}{\sin^4(\theta/2)}.$$

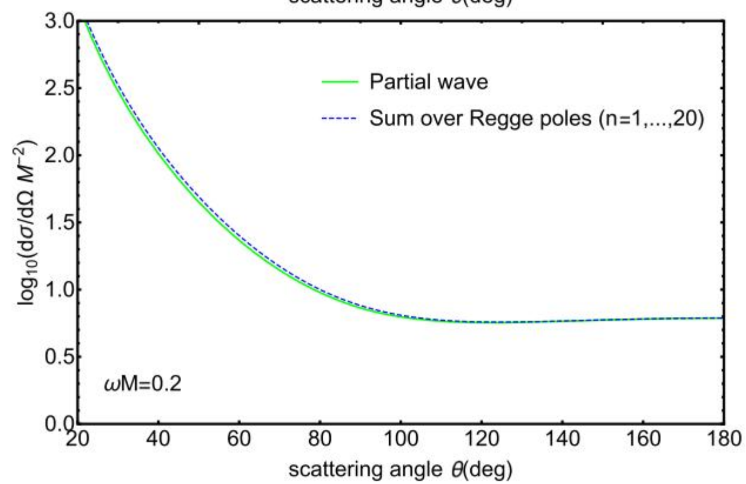


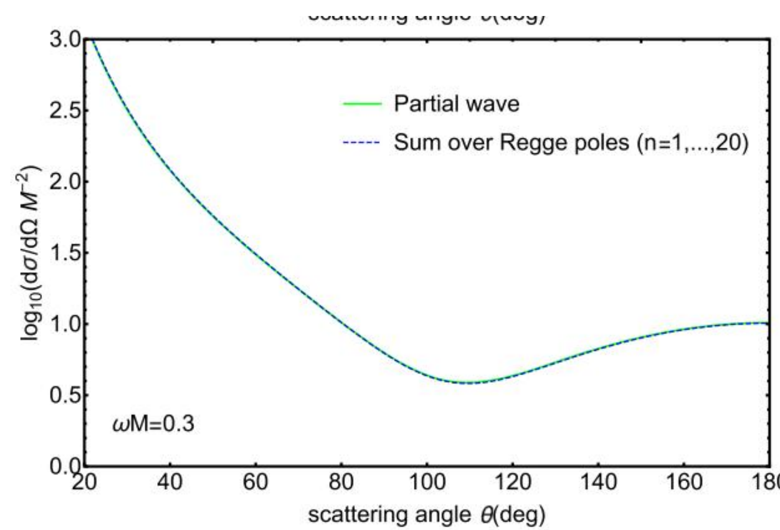
Amplitudes of the backscattered waves (θ = 180 ◦) as functions of q for different values of ωM.



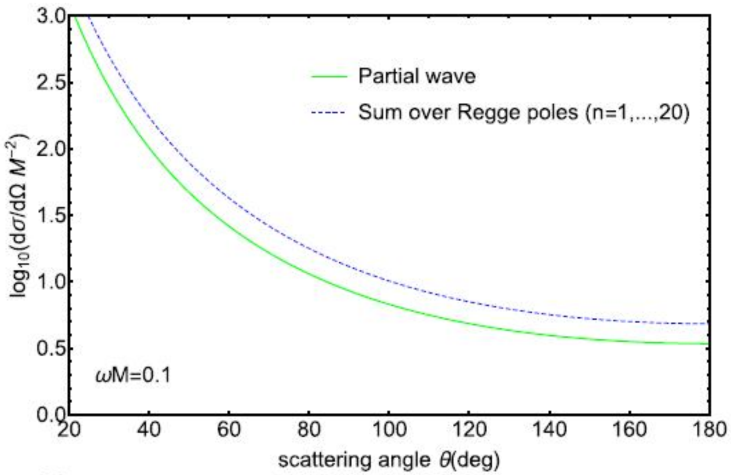
Comparison of the scattering cross sections obtained via the partial wave method and the Regge pole approximations

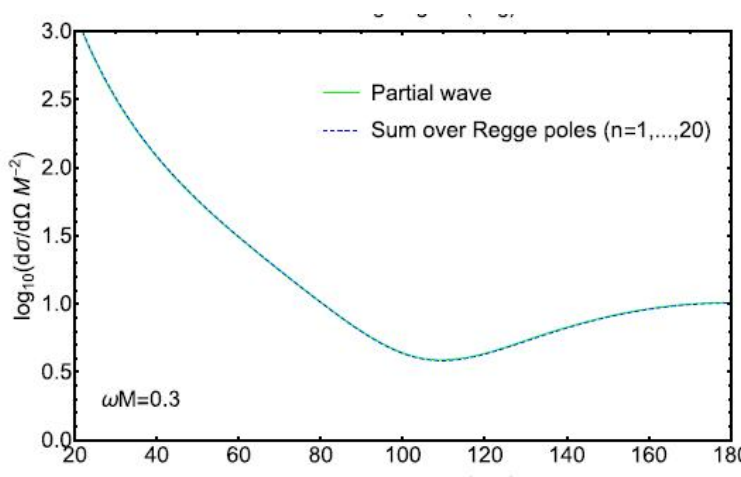


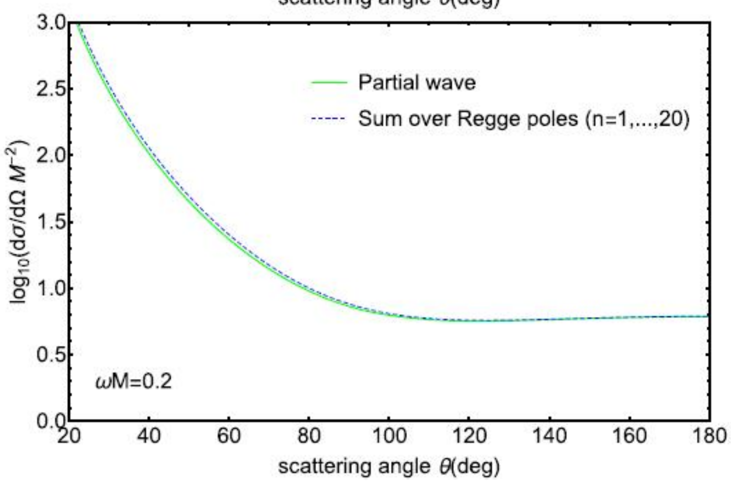


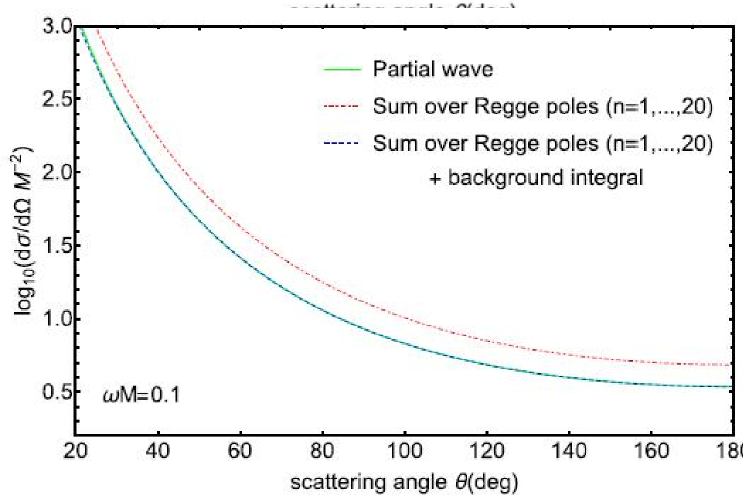


Sum over Regge poles vs partial wave (q=0)



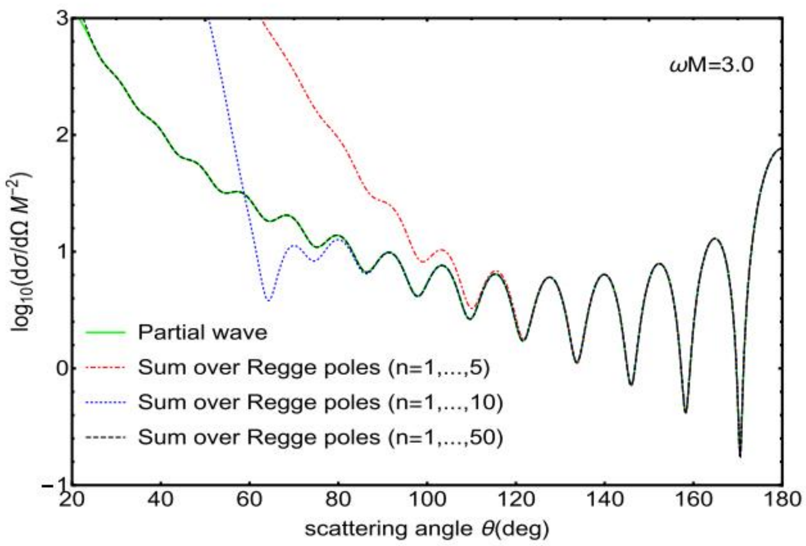




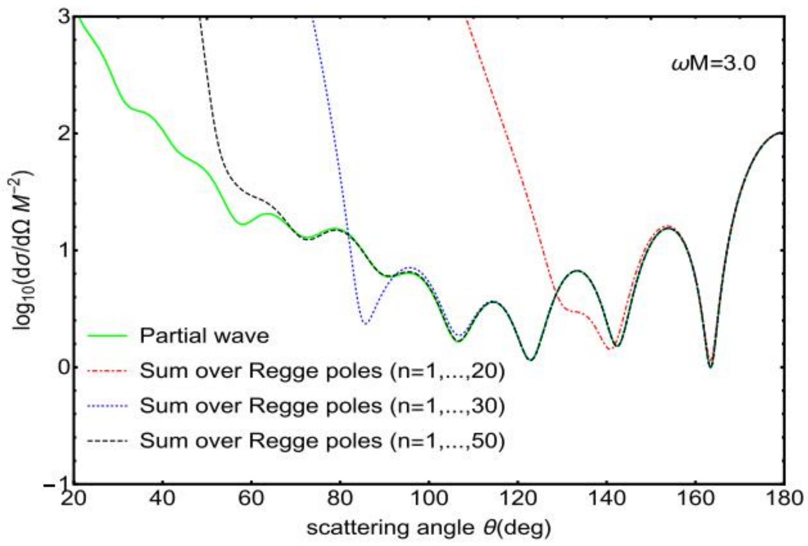


Poles required by different q

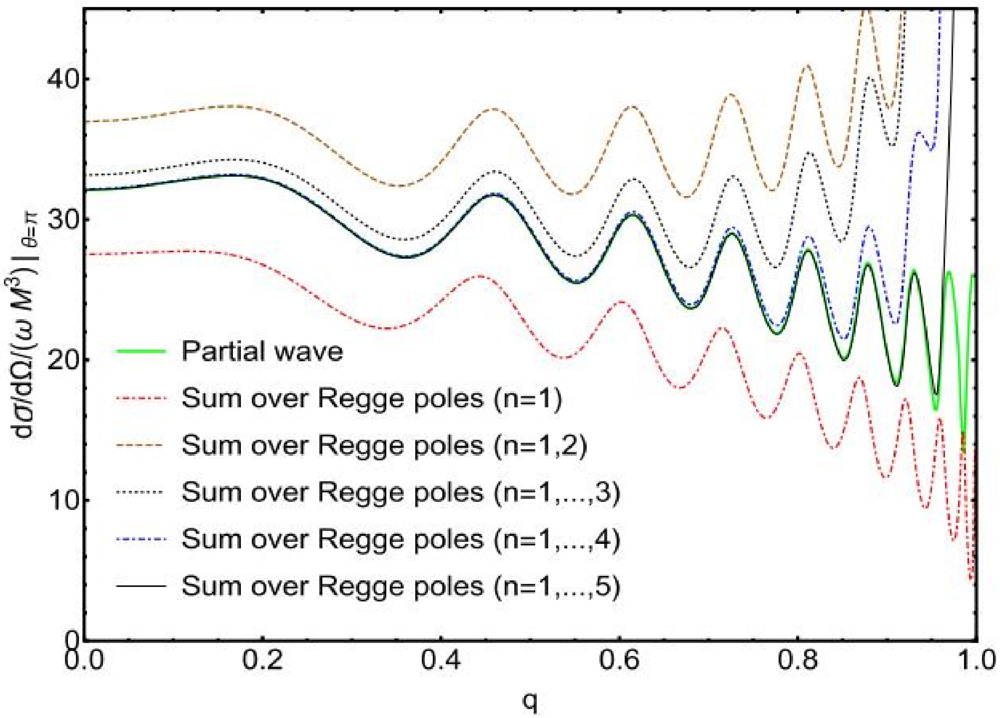
q=0.3



q=0.999

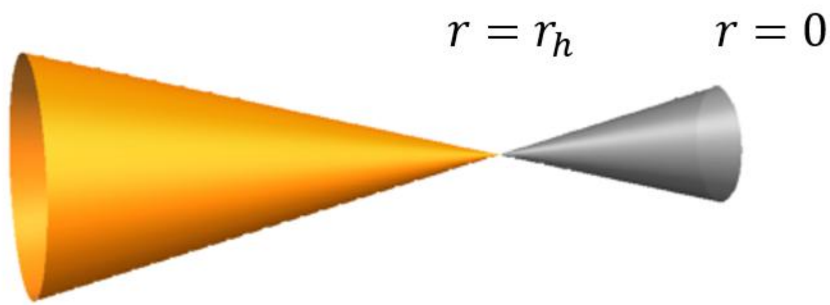


Amplitudes of the backscattered waves by the partial wave method and the Regge poles

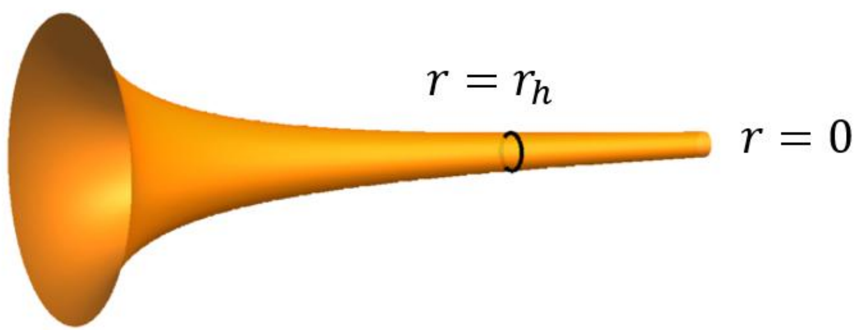


$\omega M = 3.0$

Area of the horizons

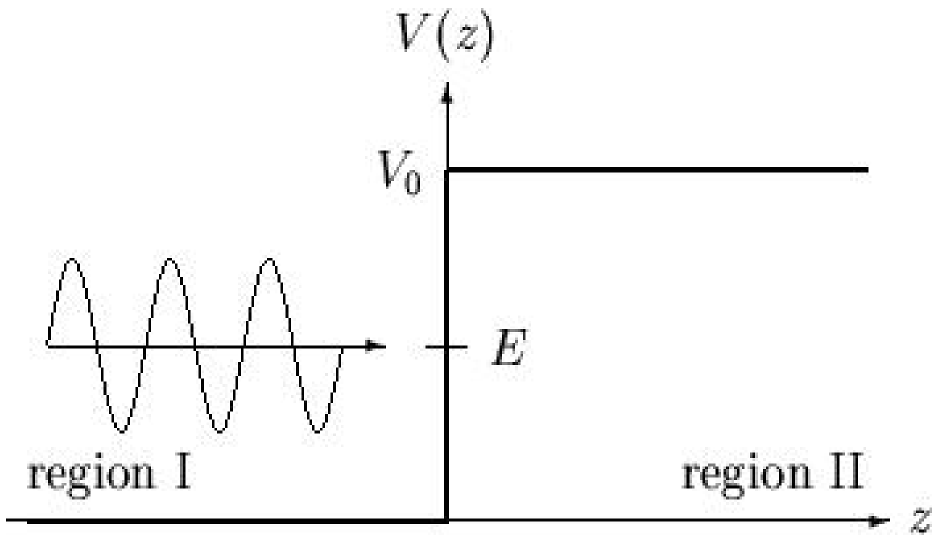


dilaton 黑洞



RN 黑洞

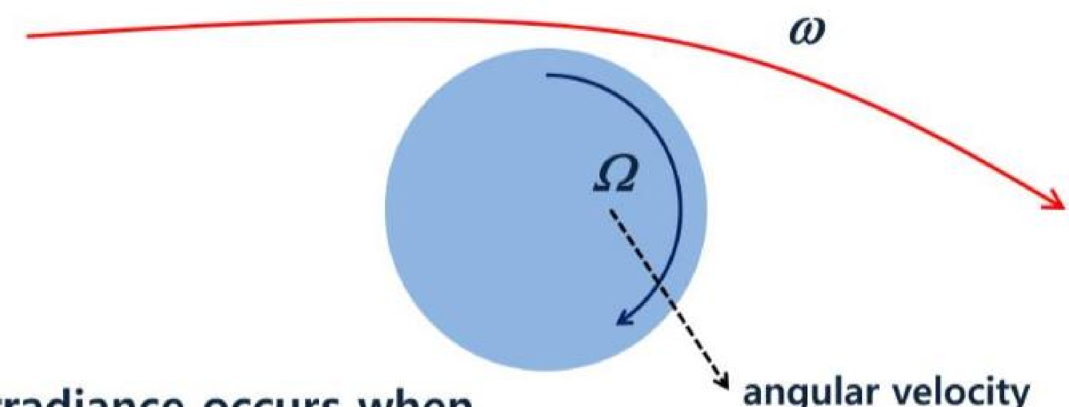
Traditional clouds: Klein mechanism



To obtain more identical particles from a potential

Superradiance: Klein mechanism in BH spacetime

Rotational energy is extracted to the scattered particle.



Superradiance occurs when

$$\omega < m\Omega$$

Traditional clouds: balance between leakage and superradiance for rotating BHs

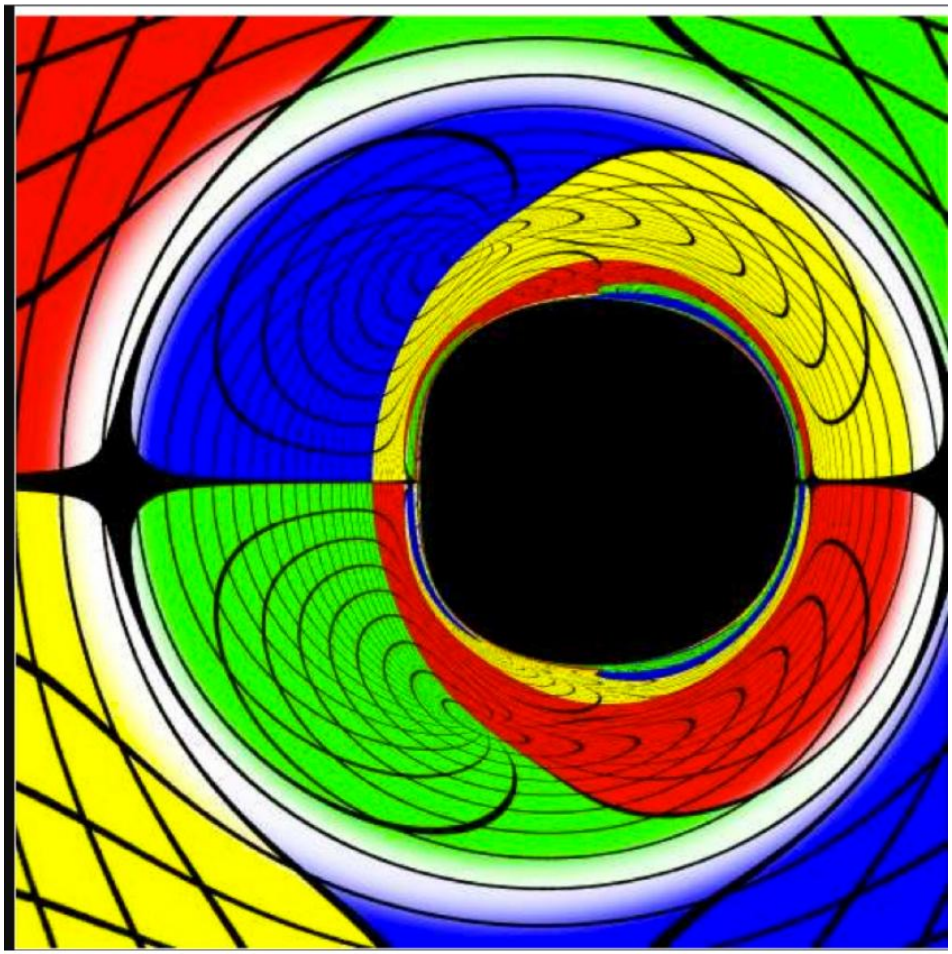


Image of a hariy BH, P Cunha et al 1509.00021

Discussions about horizon singularity

At horizon r=2m

$$R^{abcd}R_{abcd} = \frac{3072\, m^{10} - 4608\, m^8\, Q^2 + 2624\, m^6\, Q^4 - 704\, m^4\, Q^6 + 80\, m^2\, Q^8}{256\, m^6\, (2\, m^2 - Q^2)^4}$$

S.P. singularity
Not the singularity appeared in Penrose-Hawking singularity theorem
Not the singularity abhorred by Cosmic censorship

Real singularity: geodesic incomplete singularity

Geodesic analysis

- Lagrangian of a particle

$$2\mathcal{L} = -\left(1 - \frac{2M}{r}\right)\dot{t}^2 + \left(1 - \frac{2M}{r}\right)^{-1}\dot{r}^2 + r\left(r - \frac{Q^2}{M}\right)\left(\dot{\vartheta}^2 + \sin^2\vartheta\dot{\varphi}^2\right)$$

- First integral

$$\left(\frac{dr}{d\tau}\right)^2 + \left(1 - \frac{2M}{r}\right)\left[\frac{L^2}{r\left(r - \frac{Q^2}{M}\right)} - \epsilon\right] = E^2$$

- Radial particles

$$\left(\frac{dr}{d\tau}\right)^2 - \left(1 - \frac{2M}{r}\right)\epsilon = E^2$$

- Passing through the horizon smoothly.

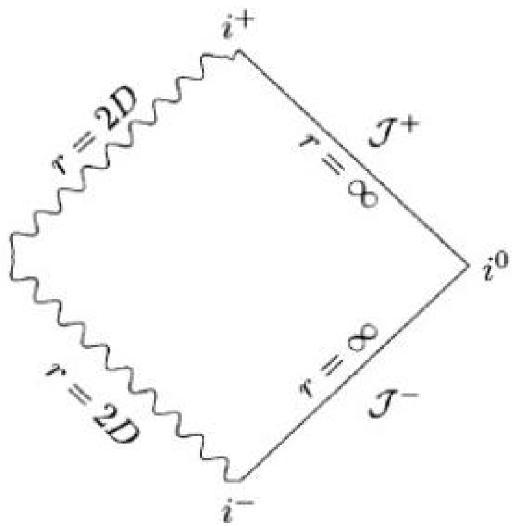
- For more details, see S. Soroushfar et al, Phys. Rev. D 94, 024010 (2016), 1601.03143

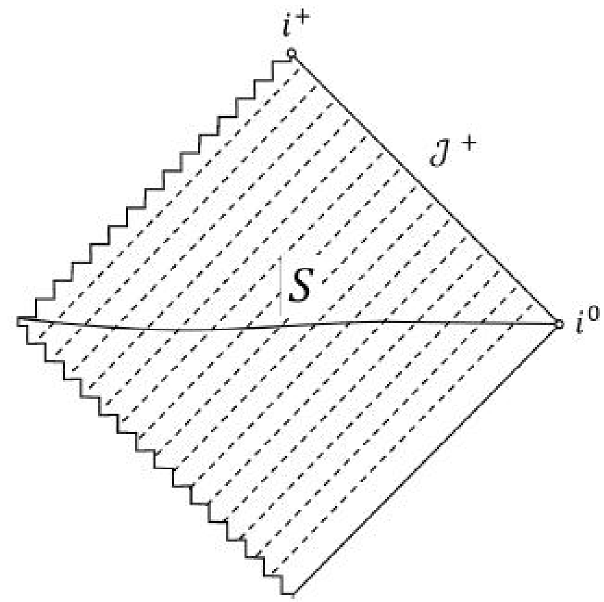
About cosmic censorship

- Still no regirous mathematical formulation.
- Essence: To guard the power of prediction of physical laws
- A formulation preferred (weak): A manifold is stongly future asymptotically predictable.
- (strong): Any timelike singularity (if exists) is invisible to any observer.

R Wald, gr-qc/9710068

Penrose diagram





The causal past of furture null infinity is globally hyperbolic.

String frame

- In string frame,

$$S = \int d^4x \sqrt{-g}\ e^{-2\phi}[-R - 4(\nabla\phi)^2 + F^2]$$

- Strings do not directly couple to metric,

but $e^{2\phi} g_{\mu\nu}$.

- Here $e^{2\phi} g_{\mu\nu}$ is finite at horizon for extreme holes, and thus it is irrelevant.

Summary

- 1.To find scalar cloud of BH independent on superradiance for the first time.
- 2.To find Dirac cloud of BH for the first time。

  3.Mechnism for the existence of clouds。

  4. Extreme dilatonic BHs behave as elementary particles。

Thank you!